Fractals and the Chaos Game

Jonas Wolter

Week 3

We have seen two different possibilities to generate fractals yet. One uses more-or-less descriptions on how a fractal is produced. The other one uses L-systems.
Interstingly, there is a third method to generate fractals using a probalistic algorithm which is known as the *Chaos Game*. As an example, the following algorithm produces the Sierpinski Triangle which we have already seen when we were looking at L-systems.

1. Draw an equilateral triangle, labelling the vertices A, B, and C.
2. Draw a point anywhere inside the triangle.
3. Choose one of A, B or C with equal probability (for example by rolling a standard die and choosing A on a roll of 1 or 2, B on 3 or 4 and C on 5 or 6).
4. Move half way from your point towards the chosen vertex and draw another point.
5. Repeatedly apply steps 3 and 4, each time starting from the point just drawn.

Exercise 1. *Why does the given algorithm costruct the Sierpinski triangle?*

Exercise 2. *Is the randomness of every step necessary for the construction of the Sierpinksi triangle? In other words: Can we generate a sequence of choice of vertices that also generates the Sierpinksi triangle?*

Exercise 3. *What happens if we start with an initial point outside the triangle? What happens if not all vertices are chosen with the same probability?*

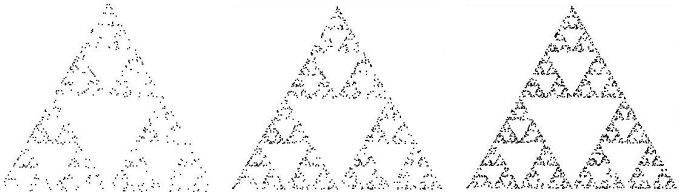

Figure 1: Chaos Game after 500, 1000 and 2000 iterations.

Seeing this quite interesting result of the Chaos game in a triangle gives rise to the question what happens in other n-gons. Unfortunately the result is not always nice and sometimes the needed values are really awful, but one pretty fasciting figure may be obtained in the following exercise.

Exercise 4. *Try to figure out the result of the Chaos game in an hexagon when one moves in every step* $\frac{1}{3}$ *of the current distance to the chosen vertex?*